Name: _____ Date: _____

Constant Rate of Change Notes

Work Zone

$\frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$

Linear Relationships

Relationships that have straight-line graphs, like the one on the previous page, are called linear relationships. Notice that as the number of songs increases by 2, the time in minutes increases by 1.

Number of Songs, y	0	2	4	6	8
Time (minutes), x	0	1	2	3	4

(+2 +2 +2 +2 over the Number of Songs row; +1 +1 +1 +1 over the Time row)

Rate of Change

$\frac{2}{1} = 2$ songs per minute

The rate of change between any two points in a linear relationship is the same or *constant*. A linear relationship has a constant rate of change.

Example

Tutor

1. The balance in an account after several transactions is shown. Is the relationship between the balance and number of transactions linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Transactions (x)	Balance ($) ($y$)
3	170
6	140
9	110
12	80

(+3, +3, +3; −30, −30, −30)

As the number of transactions increases by 3, the balance in the account decreases by $30.

Since the rate of change is constant, this is a linear relationship.
The constant rate of change is $\frac{-30}{3}$ or −$10 per transaction.
This means that each transaction involved a $10 *withdrawal*.

(a) Not a linear relationship b/c there is not a constant rate of change between the time and temp. Show your work.

(b) Yes it's linear!

a. $\frac{-20}{8} = -\frac{5}{2}$ or $-2\frac{1}{2}$

b. _____

The rate of change is −2.5 min per volunteer.

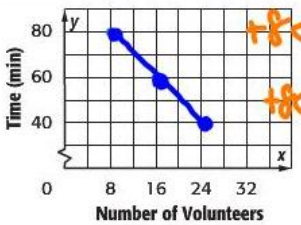
Got It? Do these problems to find out.

a.

Cooling Water	
Time (min)	Temperature (°F)
5	95
10	90
15	86
20	82

(+5, +5, +5; −5, −4, −4)

b.



(8, 80), (16, 60), (24, 40); +8, +8; −20, −20

Proportional Linear Relationships

Key Concept

Words Two quantities a and b have a proportional linear relationship if they have a constant ratio and a constant rate of change.

Symbols $\frac{b}{a}$ is constant and $\frac{\text{change in } b}{\text{change in } a}$ is constant.

To determine if two quantities are proportional, compare the ratio $\frac{b}{a}$ for several pairs of points to determine if there is a constant ratio.

Graph has to go through the origin!

Proportional Relationships
Two quantities are proportional if they have a constant ratio.

Example

Watch Tutor

2. **Use the table to determine if there is a proportional linear relationship between a temperature in degrees Fahrenheit and a temperature in degrees Celsius. Explain your reasoning.**

Degrees Celsius	0	5	10	15	20
Degrees Fahrenheit	32	41	50	59	68

+5 +5 +5 +5 (Degrees Celsius)

+9 +9 +9 +9 (Degrees Fahrenheit)

Constant Rate of Change

$$\frac{\text{change in °F}}{\text{change in °C}} = \frac{9}{5}$$

$\frac{32}{0} \neq \frac{41}{5} \neq \frac{50}{10}$

Since the rate of change is constant, this is a linear relationship.

$F = \frac{9}{5}C + 32$

To determine if the two scales are proportional, express the relationship between the degrees for several columns as a ratio.

$$\frac{\text{degrees Fahrenheit}}{\text{degrees Celsius}} \rightarrow \frac{41}{5} = 8.2 \qquad \frac{50}{10} = 5 \qquad \frac{59}{15} \approx 3.9$$

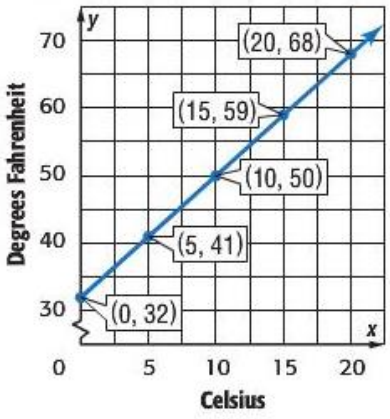
$\approx F = 2C + 30$

Since the ratios are not the same, the relationship between degrees Fahrenheit degrees Celsius is *not* proportional.

Check: Graph the points on the coordinate plane. Then connect them with a line.

The points appear to fall in a straight line so the relationship is linear. ✓

The line connecting the points does not pass through the origin so the relationship is not proportional. ✓



Got It? Do this problem to find out.

c. Use the table to determine if there is a proportional linear relationship between mass of an object in kilograms and the weight of the object in pounds. Explain your reasoning.

Weight (lb)	20	40	60	80
Mass (kg)	9	18	27	36

+20 +20 +20

+9 +9 +9

$\frac{9}{20} ? \frac{18}{40} ? \frac{27}{60} ? \frac{36}{80}$

Yes! It is a proportional linear relationship because it has a constant rate of change of 0.45 kg per lb and the ratios are equivalent.

Show your work.

Guided Practice

Check

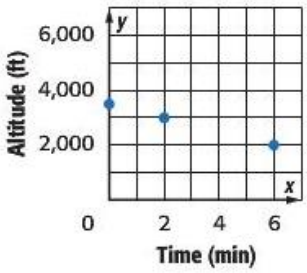
1. The amount of paint y needed to paint a certain amount of chairs x is shown in the table. Is the relationship between the two quantities linear? If so, find the constant rate of change. If not, explain your reasoning. (Example 1)

Paint Needed for Chairs	
Chairs, x	Cans of Paint, y
5	6
10	12
15	18

Show your work.

Yes; the rate of change between total cans of paint and number of chairs for each number of chairs is a constant $\frac{6}{5}$ or $1\frac{1}{5}$ cans per chair.

2. The altitude y of a certain airplane after a certain number of minutes x is shown in the graph. Is the relationship linear? If so, find the constant rate of change. If not, explain your reasoning. (Example 1)



yes; −250 ft/min or a decrease of 250 feet each minute

3. Determine whether a proportional relationship exists between the two quantities shown in Exercise 1. Explain your reasoning. (Example 2)

Yes; the ratio of cans of paint to number of chairs is a constant $1\frac{1}{5}$ cans of paint per chair so the relationship is proportional.

The rate of change is a constant $1\frac{1}{5}$ cans of paint per chair so the relationsip is linear.