Name: ____ Date: ___________________

Multiplying and Dividing Monomials

Warm-up: The metric system is the system of measurement that the vast majority of the world uses on a daily basis to find measurements for length, mass, and capacity. One amazing aspect of the metric system is the consistency of the prefixes used for the names of units in each of these three categories. For example, since there are 1000 millimeters in 1 meter, we also know there are 1000 milligrams in 1 gram and 1000 milliliters in 1 liter.

Let's use this relationship to help us discover a special relationship with powers and monomials by filling in the table below.

Unit of Measure Prefix	Times Longer than a Milli Unit	Written using Powers
Milli	1	10^0
Centi	$1 \times 10 = 10$	$10^0 \times 10^1 = 10^1$
Deci	$10 \times 10 = 100$	$10^1 \times 10^1 = 10^2$
Base Unit (m, g, L)	$100 \times 10 =$ 1000	$10^2 \times 10^1 = 10^{3}$
Deka	$1000 \times 10 =$ 10,000	$10^3 \times 10^1 = 10^{4}$
Hecto	10,000 $\times 10 =$ 100,000	$10^{4} \times 10^1 = 10^{5}$
Kilo	100,000 $\times 10 =$ 1,000,000	$10^{5} \times 10^1 = 10^{6}$

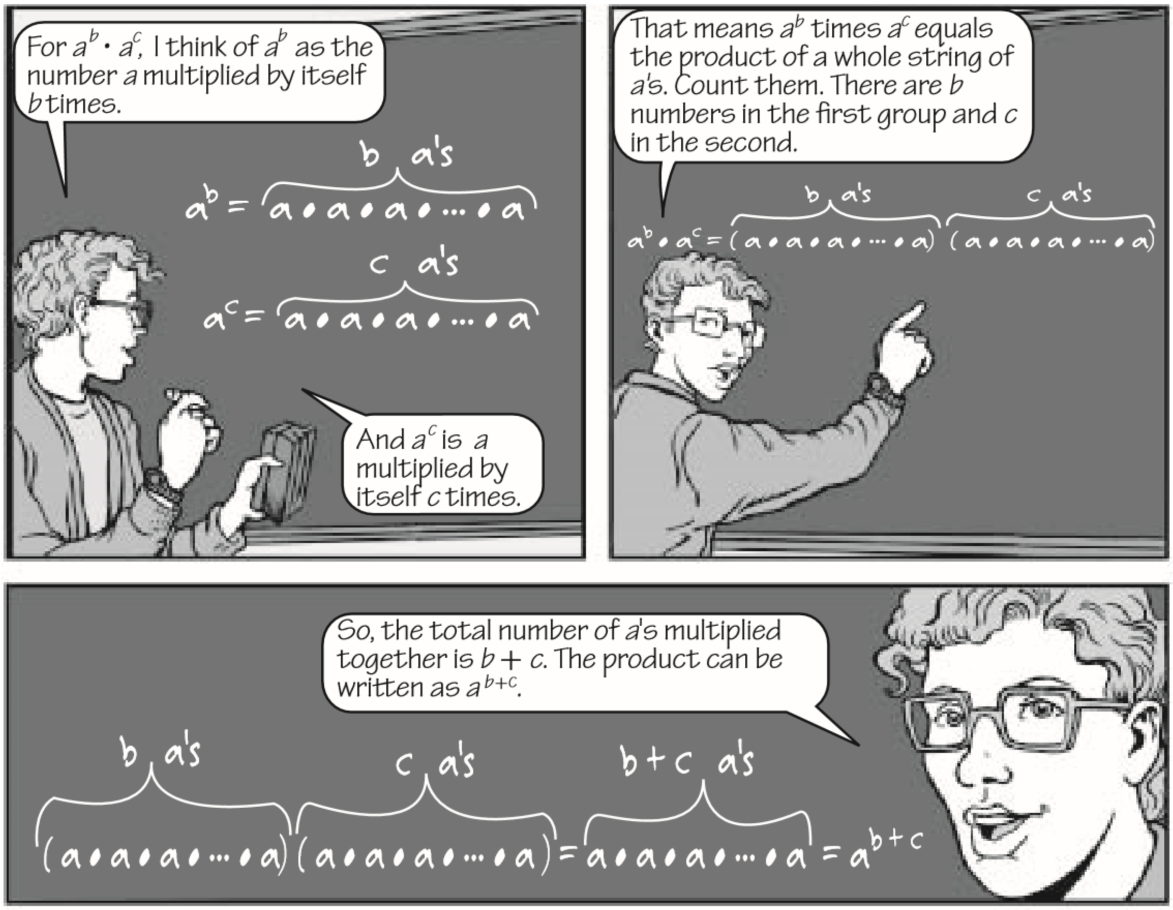
Look at the entries in the last column. What do you observe about the exponents of the factors and the exponent of the product for each entry?

The exponent of the product is the sum of the exponents of the factors.

Maybe this pattern is specific to only this scenario… try applying this pattern to the following problems:

Factors	Factors as Powers with Same Base	Product	Product as Power with Same Base as Factors
8×16	$2^3 \times 2^4$ 3+4	128	7 2^{7}
9×81	$3^{2} \times 3^{4}$ 2+4	729	6 3^{6}
125×5	$5^{3} \times 5^{1}$ 3+1	625	4 5^{4}

The pattern still worked! Crazy, right?! It worked thanks to one of the Laws of Exponents. Here's a "comical" way to describe this law:



Product of Powers

Words To multiply powers with the same base, add their exponents.

Examples

Numbers: $2^4 \cdot 2^3 = 2^{4+3}$ or 2^7

Algebra: $a^m \cdot a^n = a^{m+n}$

A **monomial** is a number, a variable, or a product of a number and one or more variables. You can use the Laws of Exponents to simplify monomials.

$$3^2 \cdot 3^4 = \underbrace{\overbrace{(3 \cdot 3)}^{\text{2 factors}} \cdot \overbrace{(3 \cdot 3 \cdot 3 \cdot 3 \cdot)}^{\text{4 factors}}}_{\text{6 factors}} \text{ or } 3^6$$

Notice that the sum of the original exponents is the exponent in the final product.

Examples

Simplify using the Laws of Exponents.

1. $5^2 \cdot 5^1 = 5^{2+1} = 5^3$

$(5 \cdot 5) \cdot 5$

$5^3 = 125$

2. $c^3 \cdot c^5$

$c^{3+5} = c^8$

3. $-3x^2 \cdot 4x^5$

$-3 \cdot x^2 \cdot 4 \cdot x^5$

$\boxed{-12x^7}$

Got It? Do these problems to find out.

a. $9^3 \cdot 9^2$

$9^{3+2} = 9^5$

b. $a^3 \cdot a^2 = 5$

a^5

c. $-2m(-8m^5)$

$m^1 \cdot m^5 = m^{1+5}$

$-2 \cdot m^1 \cdot -8 \cdot m^5$

$16 \cdot m^6$

$16m^6$

This pattern is awesome for multiplying monomials, but what if we have to divide? Let's try these problems to see if there's a similar pattern:

Dividend and Divisor	Values as Powers with Same Base	Quotient	Quotient as Power with Same Base as Factors
256 ÷ 8	$2^8 \div 2^3$	32	2^5
1296 ÷ 36	$6^4 \div 6^2$	36	6^2
343 ÷ 49	$7^3 \div 7^2$	7	7^1

Notice anything interesting?

The exponent of the quotient is the difference between the exponents of the dividend and divisor, respectively.

Quotient of Powers

Words To divide powers with the same base, subtract their exponents.

Examples

Numbers

$$\frac{3^7}{3^3} = 3^{7-3} \text{ or } 3^4$$

Algebra

$$\frac{a^m}{a^n} = a^{m-n}, \text{ where } a \neq 0$$

$$\frac{\cancel{2}^{\,1}}{\cancel{3}_{\,1}} \times \frac{\cancel{6}^{\,2}}{\cancel{10}_{\,5}}$$

$\frac{2 \div 2}{3 \div 3} \times \frac{6 \div 3}{10 \div 2}$

There is also a Law of Exponents for dividing powers with the same base.

$$\frac{5^7}{5^4} = \frac{\overbrace{5 \cdot 5 \cdot 5 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}^{\text{7 factors}}}{\underbrace{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}_{\text{4 factors}}} \text{ or } 5^3$$

Notice that the difference of the original exponents is the exponent in the final quotient.

Examples

Simplify using the Laws of Exponents.

4. $\frac{4^8}{4^2}$

$4^{8-2} = 4^6$

5. $\frac{n^9}{n^4}$

$n^{9-4} = n^5$

6. $\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5}$

$2^{5-2} = 2^3$

$3^{5-4} = 3^1$

$5^{2-1} = 5^1$

$\boxed{2^3 \cdot 3 \cdot 5}$

Got It? Do these problems to find out.

d. $\frac{5^7}{5^4}$ 5^3

e. $\frac{x^{10}}{x^3}$ x^7

f. $\frac{12w^5}{2w}$ $\boxed{6w^4}$

w^{5-1}

g. $\dfrac{3^4 \cdot 5^2 \cdot 7^5}{3^2 \cdot 5 \cdot 7^3}$

$3^2 \cdot 5 \cdot 7^2$

h. $\dfrac{5^6 \cdot 7^4 \cdot 8^3}{5^4 \cdot 7^2 \cdot 8^2}$

$5^2 \cdot 7^2 \cdot 8$

i. $\dfrac{(-2)^5 \cdot 3^4 \cdot 5^7}{(-2)^2 \cdot 3 \cdot 5^4}$

$(-2)^3 \cdot 3^3 \cdot 5^3$

Power of a Power

Words To find the power of a power, multiply the exponents.

Examples

Numbers	Algebra
$(5^2)^3 = 5^{2 \cdot 3}$ or 5^6	$(a^m)^n = a^{m \cdot n}$

You can use the rule for finding the *product* of powers to discover another Law of Exponents for finding the *power* of a power.

$$(6^4)^5 = \overbrace{(6^4)(6^4)(6^4)(6^4)(6^4)}^{\text{5 factors}}$$

$$= 6^{4+4+4+4+4}$$ Apply the rule for the product of powers.

$$= 6^{4 \cdot 5}$$

$$= 6^{20}$$

Notice that the product of the original exponents, 4 and 5, is the final power 20.

Examples

Simplify using the Laws of Exponents.

1. $(8^4)^3$

$(8^4)(8^4)(8^4)$

$= 8^{4+4+4} = 8^{12} = 8^{4 \cdot 3}$

2. $(k^7)^5$

$k^{7 \cdot 5} = k^{35}$

Got It? Do these problems to find out.

a. $(2^5)^2$

2^{10}

b. $(w^4)^6$

w^{24}

c. $[(3^2)^3]^2 = 3^{12}$

$[3^2 \cdot 3^2 \cdot 3^2]^2$

Power of a Product

Words To find the power of a product, find the power of each factor and multiply.

Examples

Numbers: $(6x^2)^3 = (6)^3 \cdot (x^2)^3$ or $216x^6$

Algebra: $(ab)^m = a^m b^m$

Extend the power of a *power* rule to find the Laws of Exponents for the power of a *product*.

$$(3a^2)^5 = \overbrace{(3a^2)(3a^2)(3a^2)(3a^2)(3a^2)}^{\text{5 factors}}$$

$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot a^2 \cdot a^2 \cdot a^2 \cdot a^2 \cdot a^2$$

$$= 3^5 \cdot (a^2)^5 \quad \text{Write using powers.}$$

$$= 243 \cdot a^{10} \text{ or } 243a^{10} \quad \text{Power of a Power}$$

Common Error

When finding the power of a power, do not add the exponents.

$(8^4)^3 = 8^{12}$, not 8^7.

Examples

Simplify using the Laws of Exponents.

3. $(4p^3)^4$

$(4p^3)(4p^3)(4p^3)(4p^3)$

$4^4 \cdot (p^3)^4$

$256 \cdot p^{3 \cdot 4} = \boxed{256p^{12}}$

4. $(-2m^7n^6)^5$

$(-2)^5 \cdot (m^7)^5 \cdot (n^6)^5$

$-32 \cdot m^{35} \cdot n^{30}$

$\boxed{-32m^{35}n^{30}}$

Got It? Do these problems to find out.

d. $(8b^9)^2$

$64b^{18}$

e. $(6x^5y^{11})^4$

$1296x^{20}y^{44}$

f. $(-5w^2z^8)^3$

$-125w6z^{24}$