

Rotations

Rotate a Figure About a Point

A **rotation** is a transformation in which a figure is rotated, or turned, about a fixed point. The **center of rotation** is the fixed point. A rotation does not change the size or shape of the figure. So, the preimage and the image are congruent.

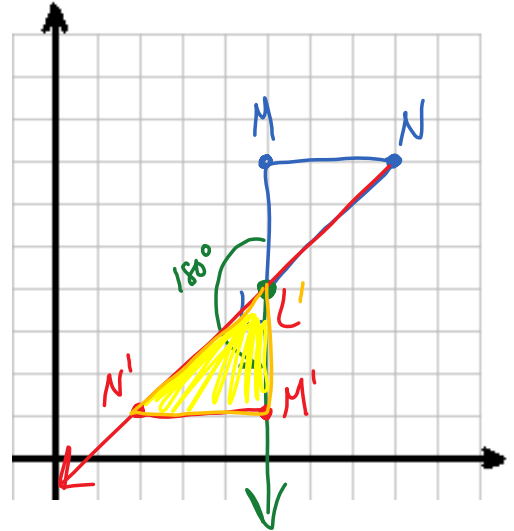


Example



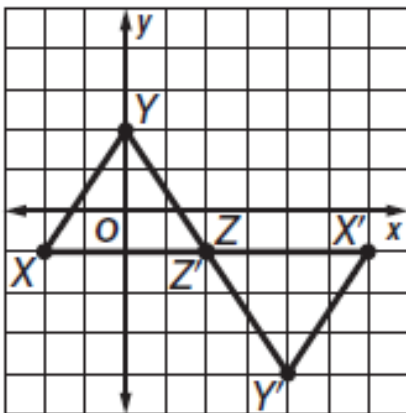
- Triangle LMN with vertices $L(5, 4)$, $M(5, 7)$, and $N(8, 7)$ represents a desk in Jackson's bedroom. He wants to rotate the desk counterclockwise 180° about vertex L . Graph the figure and its image. Then give the coordinates of the vertices for $\triangle L'M'N'$.

$L'(5, 4)$
 $M'(5, 1)$
 $N'(2, 1)$



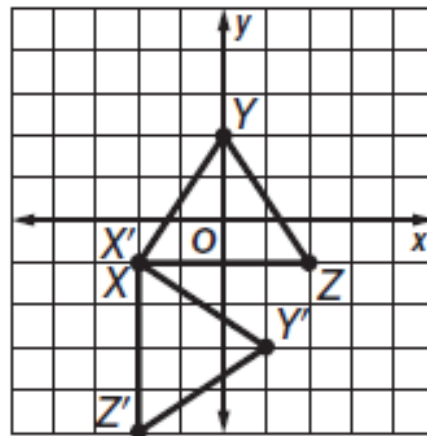
Now you try! For Exercises 1 and 2, graph $\triangle XYZ$ and its image after each rotation. Then give the coordinates of the vertices for $\triangle X'Y'Z'$.

- a. 180° clockwise about vertex Z



$X'(6, 1)$, $Y'(4, -4)$, $Z'(2, -1)$

- b. 90° clockwise about vertex X

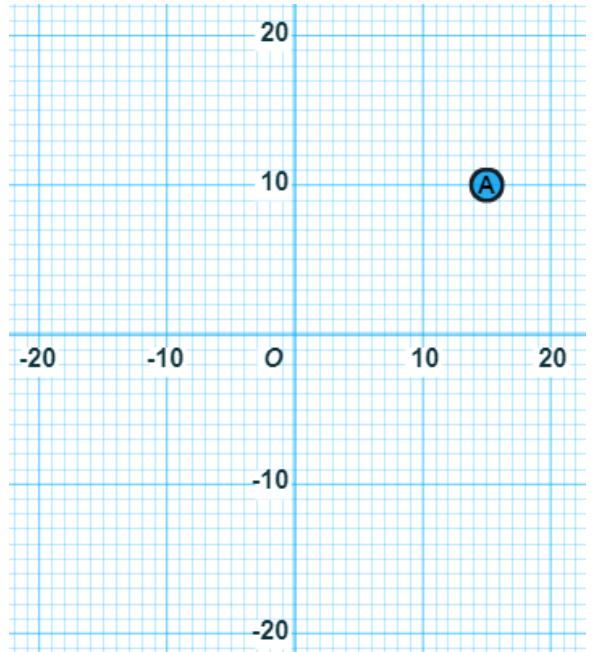


$X'(-2, -1)$, $Y'(1, -3)$, $Z'(-2, -5)$

How do you rotate geometric figures about the origin?

Point A is located at (15, 10) on the coordinate plane to the right. Point B will be the rotation of Point A about the origin.

Plot point B after a 90° , 180° , 270° , and 360° rotation and label the rotations on the coordinate plane. Then, provide the coordinates of point B at each rotation in the table below.



1. What direction are the points moving in as the degrees of rotation increases? Decreases?

↓
Counter-clockwise (+) ; Clockwise (-)

2. What angle of rotation brings point B back to point A?

360°

3. Provide the coordinates of point B at each rotation in the table below. Then, determine if you see any patterns with the ordered pairs and point A's coordinates.

Rotation Angle	Coordinates
$90^\circ/-270^\circ$	$(-10, 15)$
$180^\circ/-180^\circ$	$(-15, -10)$
$270^\circ/-90^\circ$	$(10, -15)$
360°	$(15, 10)$

$(15, 10)$

4. Based on the patterns that you have observed, write the general coordinates of the image of a point rotated about the origin with coordinates (x, y) in the table below.

Angle of rotation	0°	$90^\circ/-270^\circ$	$180^\circ/-180^\circ$	$270^\circ/-90^\circ$	360°
Coordinates of image of (x, y)	(x, y)	$(-y, x)$	$(-x, -y)$	$(y, -x)$	(x, y)

Example



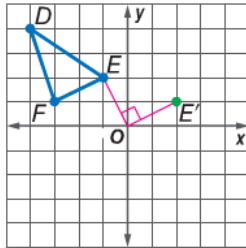
- 2.** Triangle DEF has vertices $D(-4, 4)$, $E(-1, 2)$, and $F(-3, 1)$. Graph the figure and its image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices for $\triangle D'E'F'$.

$$(y, -x)$$

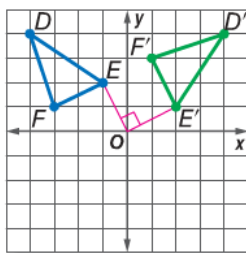
$$D'(4, 4) \quad F'(1, 3)$$

$$E'(2, 1)$$

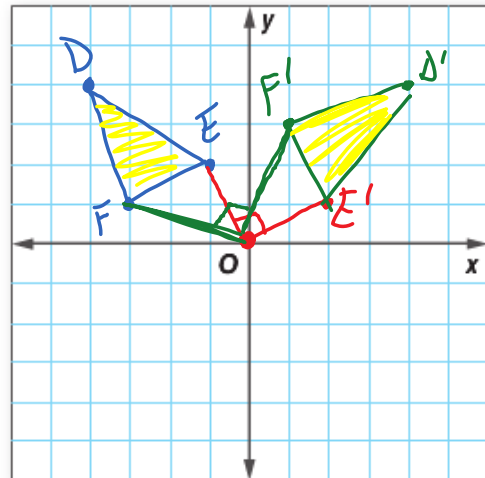
Step 1 Graph $\triangle DEF$ on a coordinate plane.



Step 2 Sketch segment \overline{EO} connecting point E to the origin. Sketch another segment, $\overline{E'O}$, so that the angle between point E , O , and E' measures 90° and the segment is the same length as \overline{EO} .



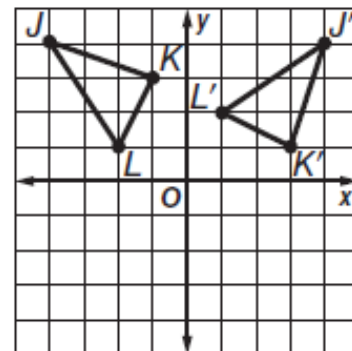
Step 3 Repeat Step 2 for points D and F . Then connect the vertices to form $\triangle D'E'F'$.



So, the coordinates of the vertices of $\triangle D'E'F'$ are $D'(4, 4)$, $E'(2, 1)$, and $F'(1, 3)$.

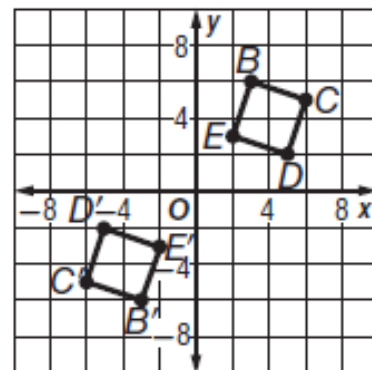
Now you try!

- a.** Triangle JKL has vertices $J(-4, 4)$, $K(-1, 3)$, and $L(-2, 1)$. Graph the figure and its rotated image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices for triangle $J'K'L'$.



$$J'(4, 4), K'(3, 1), \text{ and } L'(1, 2)$$

- b.** Quadrilateral $BCDE$ has vertices $B(3, 6)$, $C(6, 5)$, $D(5, 2)$, and $E(2, 3)$. Graph the figure and its rotated image after a counterclockwise rotation of 180° about the origin. Then give the coordinates of the vertices for quadrilateral $B'C'D'E'$.



$$B'(-3, -6), C'(-6, -5), D'(-5, -2), \text{ and } E'(-2, -3)$$